# Flexible Aggregation with SUOWA Operators in R

## An Implementation based on the Choquet Integral

T. Gonzalez-Arteaga[1] B. Llamazares[2] R. de Andres Calle[3]

[1] Department of Statistic and O.R., Universidad de Valladolid [2] Instituto de Matemáticas (IMUVA), Universidad de Valladolid
[3] Multidisciplinary Institute of Enterprise (IME), Universidad de Salamanca

## From Theory to Software

WEMOWA bridges theoretical developments in generalized aggregation operators, SUOWA, with practical tools for decision making and methodological research [2] [1]

### Context

In many decision-making and information aggregation problems, two complementary aspects must be considered simultaneously:

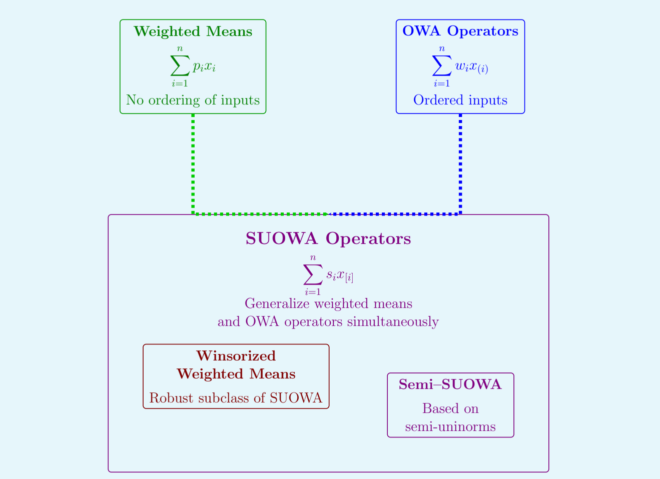
- The importance of each information source or criterion
- The importance of the relative position of the values (ordering)

Classical aggregation operators address only one of these aspects. SUOWA operators overcome this limitation by integrating source weights and ordered values into a single aggregation operator

| Operator | Source weights | Ordered values |
|---|---|---|
| Weighted Mean | ✓ | × |
| OWA Operator | × | ✓ |
| **SUOWA Operator** | ✓ | ✓ |

## 1 SUOWA operators

### Scheme



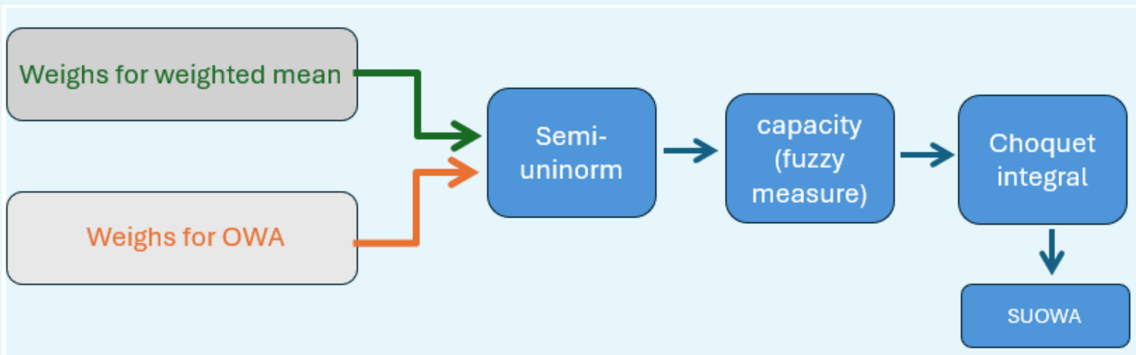

### SUOWA operators

- It is particularly useful in multi-criteria decision-making
- It retains the advantages of the weighted means and the OWA operators
- It allows the importance of each criterion and the influence of the relative position of the values to be combined into a single operator
- The SUOWA operator: $S^{U}_{p,w}(x) = \sum_{i=1}^{n} s_i x_{[i]}$, where $s_i = \hat{\nu}^{U}_{p,w}(A_{[i]}) - \hat{\nu}^{U}_{p,w}(A_{[i-1]})$ and $A_{[i]} = \{[1], \ldots, [i]\}\ \forall i \in \mathbb{N}$

## 2 WEMOWA: An R Package for Winsorized Weighted Means and SUOWA Aggregation

### What is genuinely new?

- Construction of the capacities associated with SUOWA and Semi-SUOWA operators,
- Computation of the corresponding Choquet integrals with its indices: the orness, the Shapley values, the veto, favor, k-conjunctiveness and k-disjunctiveness indices.
- Implementation of winsorized weighted means, a practically important subclass of SUOWA operators




## Acknowledgment

The authors acknowledge financial support through the research project PID2022-139469NB-I00, funded by the Spanish MCIN/AEI/ 10.13039/501100011033 and by ERDF

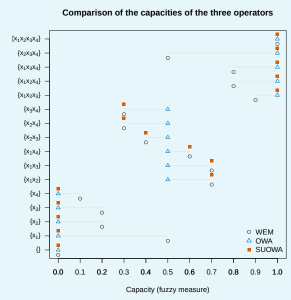
## 3 Example of use

### Aggregation problem

Consider 4 sensors measuring the same physical quantity. They have different levels of reliability, weights $p = (0.5, 0.2, 0.2, 0.1)$ To reduce the influence of a faulty sensor, the OWA weighting vector $w = (0, 0.5, 0.5, 0)$, so the minimum and maximum readings are discarded

Sensor readings for four trial

| | $t_1$ | $t_2$ | $t_3$ | $t_4$ |
|---|---|---|---|---|
| Sensor 1 | 10 | 4 | 4 | 2 |
| Sensor 2 | 5 | 5 | 10 | 10 |
| Sensor 3 | 6 | 6 | 5 | 5 |
| Sensor 4 | 4 | 10 | 2 | 4 |




Aggregation results

| | $t_1$ | $t_2$ | $t_3$ | $t_4$ |
|---|---|---|---|---|
| WEM | 7.6 | 5.2 | 5.2 | 4.4 |
| OWA | 5.5 | 5.5 | 4.5 | 4.5 |
| SUOWA | 5.7 | 5.3 | 4.4 | 4.4 |

Example of computations:
$5.7 = 0\cdot(10) + 0.7\cdot(6) + 0.3\cdot(5) + 0\cdot(4)$
$5.3 = 0\cdot(10) + 0.3\cdot(6) + 0.7\cdot(5) + 0\cdot(4)$

## 4 Key Functions in WEMOWA

### Core Aggregation Functions

`SUOWA_capacity(p, w, uninorm)`
Computes the SUOWA capacity from weighting vectors and a uninorm.

`SUOWA(x, p, w, uninorm)`
Calculates the SUOWA aggregation of a set of values

`winsorized_weighted_mean()`
Computes Winsorized weighted means for robust aggregation

`weights_from_Shapley_values()`
Derives weighting vectors from Shapley importance values

### Supporting Utilities

`uninorm()`
Construction and evaluation of uninorm operators

`orness(),veto(),favor()`
Measures characterizing aggregation behavior

`summary()`
Summary methods for WEMOWA objects

`CheckMonotonicity()`
Checks whether a game satisfies the monotonicity property required for a capacity

## Github

https://github.com/Teresa-GA/WEMOWA

## References

[1] B. Llamazares. "An analysis of Winsorized weighted means". In: *Group Decision and Negotiation* 28.5 (2019), pp. 907–933.

[2] B. Llamazares. "SUOWA operators: A review of the state of the art". In: *International Journal of Intelligent Systems* 34.5 (2019), pp. 790–818.