

Flexible Aggregation with SUOWA Operators in R

An Implementation based on the Choquet Integral



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From Theory to Software

WEMOWA bridges theoretical developments in generalized aggregation operators, SUOWA, with practical tools for decision making and methodological research [2] [1]

Context

In many decision-making and information aggregation problems, two complementary aspects must be considered simultaneously:

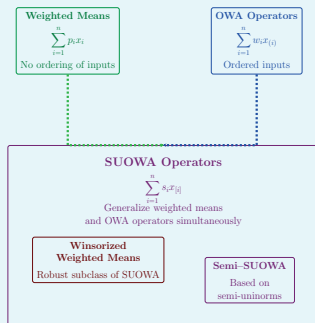
- The importance of each information source or criterion
- The importance of the relative position of the values (ordering)

Classical aggregation operators address only one of these aspects. SUOWA operators overcome this limitation by integrating source weights and ordered values into a single aggregation operator

Operator	Source weights	Ordered values
Weighted Mean	✓	✗
OWA Operator	✗	✓
SUOWA Operator	✓	✓

1 SUOWA operators

Scheme



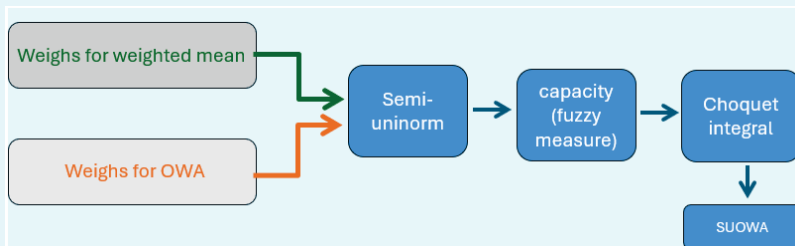
SUOWA operators

- It is particularly useful in multi-criteria decision-making
- It retains the advantages of the weighted means and the OWA operators
- It allows the importance of each criterion and the influence of the relative position of the values to be combined into a single operator
- The SUOWA operator: $S_{p,w}^U(x) = \sum_{i=1}^n s_i x_{[i]}$, where $s_i = \hat{v}_{p,w}^U(A_{[i]}) - \hat{v}_{p,w}^U(A_{[i-1]})$ and $A_{[i]} = \{[1], \dots, [i]\} \forall i \in \mathbb{N}$

2 WEMOWA: An R Package for Winsorized Weighted Means and SUOWA Aggregation

What is genuinely new?

- Construction of the capacities associated with SUOWA and Semi-SUOWA operators,
- Computation of the corresponding Choquet integrals with its indices: the orness, the Shapley values, the veto, favor, k-conjunctiveness and k-disjunctiveness indices.
- Implementation of winsorized weighted means, a practically important subclass of SUOWA operators



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3 Example of use

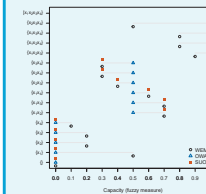
Aggregation problem

Consider 4 sensors measuring the same physical quantity. They have different levels of reliability, weights $p = (0.5, 0.2, 0.2, 0.1)$. To reduce the influence of a faulty sensor, the OWA weighting vector $w = (0, 0.5, 0.5, 0)$, so the minimum and maximum readings are discarded

Sensor readings for four trial

	t_1	t_2	t_3	t_4
Sensor 1	10	4	4	2
Sensor 2	5	5	10	10
Sensor 3	6	6	5	5
Sensor 4	4	10	2	4

Comparison of the capacities of the three operators



Aggregation results

	t_1	t_2	t_3	t_4
WEM	7.6	5.2	5.2	4.4
OWA	5.5	5.5	4.5	4.5
SUOWA	5.7	5.3	4.4	4.4

Example of computations:

$$5.7 = 0 \cdot (10) + 0.7 \cdot (6) + 0.3 \cdot (5) + 0 \cdot (4)$$

$$5.3 = 0 \cdot (10) + 0.3 \cdot (6) + 0.7 \cdot (5) + 0 \cdot (4)$$

4 Key Functions in WEMOWA

Core Aggregation Functions

`SUOWA_capacity(p, w, uninorm)`
Computes the SUOWA capacity from weighting vectors and a uninorm.

`SUOWA(x, p, w, uninorm)`
Calculates the SUOWA aggregation of a set of values

`winsorized_weighted_mean()`
Computes Winsorized weighted means for robust aggregation

`weights_from_Shapley_values()`
Derives weighting vectors from Shapley importance values

Supporting Utilities

`uninorm()`
Construction and evaluation of uninorm operators

`orness()`, `veto()`, `favor()`
Measures characterizing aggregation behavior

`summary()`
Summary methods for WEMOWA objects

`CheckMonotonicity()`
Checks whether a game satisfies the monotonicity property required for a capacity

Github

<https://github.com/Teresa-GA/WEMOWA>



References

- [1] B. Llamazares. "An analysis of Winsorized weighted means". In: *Group Decision and Negotiation* 28.5 (2019), pp. 907–933.
- [2] B. Llamazares. "SUOWA operators: A review of the state of the art". In: *International Journal of Intelligent Systems* 34.5 (2019), pp. 790–818.